Linear Algebra Previous Year Questions

Unit-1-Vector Spaces

Part-A

1. Determine whether the vectors $v_1 = (1, -2, 3), v_2 = (5, 6, -1)$ and $v_3 = (3, 2, 1)$ form a linearly dependent or independent set.
2. Does a line passing through origin in $\mathbb{R}^3$ form a subspace of $\mathbb{R}^3$?
3. Is the set $\{A \mid det(A) = 0\}$ a subspaces of the matrix M_{mxn}? Justify.
4. Why $u_1 = (-1, 2, 4), u_2 = (5, -10, -20)$ in R^3 are linearly dependent? Explain it.
5. Check whether $S = \{(x, y, 0) \mid x, y \in \mathbb{R}\}$ is a subspace of $\mathbb{R}^3$.
6. For which values of k will be vector $v = (1, -2, k)$ in R^3 be a linear combination of the vectors $u = (3, 0, -2)$ and $w = (2, -1, -5)$.
7. Check $W = \{(a_1, a_2, a_3) \mid 2a_1 - 7a_2 + a_3 = 0\}$ is subspace.
8. Find $dim\ W$ where $W = \{(x_1, x_2, x_3)/x_1 + x_2 + x_3 = 0\}$.
9. Determine whether the vectors $u = (1,1,2), v = (1,0,1), w = (2,1,3)$ span the vector space R^3.
10. Is a set of all vectors of the form $(a, 1,1)$, where a is real, a subspace of R^3.
11. Define linear independence.
12. Let v_1 and v_2 be subspaces of a vector space V. Is $v_1 \cup v_2$ a subspace of V?
13. Let V be the vector space of functions from R. Then show that the functions form R into R. Then show that the function $f(t) = sint, g(t) = e^t$ and $h(t) = t^2$ are linearly independent.
14. Using the inner product $< u, v >= 2u_1v_1 + 3u_2v_2$ to compute $< u + v, w >$ if $u = (1,1), v = (3,2)$ and $w = (0, -1)$.

Part-B

1. Show that the set V of al $2x2$ matrices with real entries is a vector space if addition is defined to be matrix addition and scalar multiplication is defined to be matrix scalar multiplication.
2. Find a basis for the space spanned by the vectors $v_1 = (1, -2,0,0,3), v_2 = (2, -5, -3, -2,6)$, $v_3 = (0,5,15,10,0)$ and $v_4 = (2,6,18,8,6)$.
3. Let $v_1 = (1,2,1)$, $v_2 = (2,9,0)$ and $v_3 = (3,3,4)$. Show that the set $S = \{v_1, v_2, v_3\}$ is basis for R^3.

4. Check whether the set of all pairs of real numbers of the form $(1, x)$ with the operations $(1, x_1) + (1, x_2) = (1, x_1 + x_2)\ and\ k(1, x) = (1, kx)$ is vector space.
5. Consider the vectors $u = (1,2,-1)$ and $v = (6,4,2)$ in R^3. Show that $w = (9,2,7)$ is a linear combination of u and v.
6. Show that the vectors $v_1 = (1,2,1)$, $v_2 = (2,9,0)$, $v_1 = (3,3,4)$ form a basis for R^3.
7. Find the dimension and basis for the solution space W of the system of homogeneous equation given below. $x_1 + 2x_2 + 2x_3 - x_4 + 3x_5 = 0$; $x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 0$; $3x_1 + 6x_2 + 8x_3 + x_4 + 5x_5 = 0$.
8. Let $V = R^3$ and $S = \{(1,0,0), (2,2,0), (5,7,2)\}$. Show that S is a minimal generating set.
9. Verify whether the set $S = \left\{\begin{pmatrix} 1 & -3 & 2 \\ -4 & 0 & 5 \end{pmatrix}, \begin{pmatrix} -3 & 7 & 4 \\ 6 & -2 & -7 \end{pmatrix}, \begin{pmatrix} -2 & 3 & 11 \\ -4 & -3 & 2 \end{pmatrix}\right\}$ in $M_{2x3}(R)$ is linearly dependent or not.
10. Determine the given set in $P_4(R)$ is linearly dependent or linearly independent for $x^4 - x^3 + 5x^2 - 8x + 6, -x^4 + x^3 - 5x^2 + 5x - 3,\ x^4 + 3x^2 - 3x + 5$ and $2x^4 + x^3 + 4x^2 + 8x$.
11. Let $S = \{v_1, v_2, v_3\}$ where $v_1 = (1,-3,-2), v_2 = (-3,1,3), v_3 = (-2,10,-2)$. Verify whether S forms a basis or not.
12. Verify whether the first polynomial can be expressed as a linear combination of the other two in $P_3(R)$ for the given $x^3 - 8x^2 + 4x,\ x^3 - 2x^2 + 3x - 1\ and\ x^3 - 2x + 3$.
13. Let W_1 and W_2 be subspaces of V. Prove that $W_1 \cup W_2$ is a subspace of V if and only if $W_1 \subseteq W_2$ (or) $W_2 \subseteq W_1$.
14. Determine whether the set of all pairs of real numbers (x, y) with the operations $(x, y) + (p, q) = (x + p + 1, y + q + 1)$ and $k(x, y) = (kx, ky)$ is a vector space or not. If not, list all the axioms that fail to hold.
15. Determine the basis and the dimension of the homogeneous system $2x_1 + 2x_2 - x_3 + x_5 = 0$; $-x_1 + -x_2 + 2x_3 - 3x_4 + x_5 = 0$; $x_1 + x_2 - 2x_3 - x_5 = 0$; $x_3 + x_4 + x_5 = 0$.
16. Prove that any intersection of subspaces of a vector space V is a subspace of V.
17. Prove that the span of any subset S of a vector space V is a subspace of V.
18. Let V be a vector space and let $S_1 \subseteq S_2 \subseteq V$. If S_2 is linearly independent, then prove that S_1 is linearly independent.
19. Let V be a vector space with dimension n. Prove that any linearly independent subset of V that contains exactly n vectors is a basis for V.

Unit-2-Linear Transformations and Diagonalization

Part-A

1. State the Dimension Theorem in linear algebra.
2. Define Identity Transformation.
3. If $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$, find eigenvalues of A^3 and A^{-1}.
4. Find the matrix representation of a linear transformation $T: P_3(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ defined by $T(f(x)) = f'(x)$ with respect to the standard ordered basis for $P_3(\mathbb{R})$ and $P_2(\mathbb{R})$.
5. Check linearity: $T: \mathbb{R}^3 \rightarrow \mathbb{R}^2$ with $T(1,0,3) = (1,1), T(-2,0,-6) = (2,1)$.
6. Let $T: P_3(\mathbb{R}) \rightarrow P_2(\mathbb{R})$ be a linear transformation defined by $T(f(x)) = f'(x)$. Let B_1 and B_2 be the standard basis for $P_3(\mathbb{R})$ and $P_2(\mathbb{R})$ respectively . Then find $[T]$.
7. Test $A = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ for diagonalizability.
8. Find kernel and range of identity operator.
9. Let $T: R^2 \rightarrow R^2$ be defined by $T(a_1, a_2) = (2a_1 + a_2, a_2)$. Is T linear?
10. Define range of linear transformation.

Part-B

1. Find the basis for the null space of $\begin{pmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$.
2. Let $T: R^2 \rightarrow R^3$ be the linear transformation defined by $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{pmatrix}$. Find the matrix for the transformation T with respect to the basis $B = \{u_1, u_2\}$ for R^2 and $B' = \{v_1, v_2, v_3\}$ for R^3, where $u_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}, u_2 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}, v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, v_2 = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}, v_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$.
3. Show that the transformation $T: R^3 \rightarrow R^2$ defined by $T(x, y, z) = (z, x + y)$ is linear.
4. Find a basis and dimension of $R(T)$ and $N(T)$ for the linear transformation $T: R^3 \rightarrow R^3$ defined by $T(a, b, c) = (a - b + 2c, 2a + b, -a - 2b + 2c)$.
5. Let T be a linear operator on $P_2(R)$ defined by $T(f(x) = f(1) + xf'(0) + [f'(0) + f''(0)]x^2$. Test for diagonalizability.

6. Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$.

7. Reduce the matrix $A = \begin{pmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{pmatrix}$ to the diagonal form.

8. Let $T: R^3 \to R^2$ be defined by $T(x, y, z) = (2x, -y, 3z)$, Verify whether $T$is linear or not. Find $N(T)$ and $R(T)$and hence verify the dimension theorem.

9. Let $P_2(R \to P_2(R)$ be defind as $T\{f(x)\} = f(x) + (x+1)f'(x)$. Find eigen values and corresponding eigenvectors of T with respect to standard basis of $P_2(R)$.

10. Test for diagonalizability of the matrix A = $\begin{bmatrix} 7 & -4 & 0 \\ 8 & -5 & 0 \\ 6 & -6 & 0 \end{bmatrix}$ and if A is diagonalizable, find the invertible matrix Q such that $Q^{-1}AQ = D$.

11. Let T be the linear operator on R^3 defined by T$\begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} = \begin{bmatrix} 4a_1 + a_3 \\ 2a_1 + 3a_2 + 2a_3 \\ a_1 + 4a_3 \end{bmatrix}$. Determine the eigenspace of T corresponding to each eigenvalue. Let B be the standard ordered basis for R^3.

Unit-3-Inner Product Spaces

Part-A

1. Find the angle between $u = (4, 3, 1, -2)$ and $v = (-2, 1, 2, 3)$.
2. Prove that: $\| \alpha + \beta \| \leq \| \alpha \| + \| \beta \|$ for any two vectors α, β belong to the standard inner product space.
3. Find orthogonal complement of $S = (0,0,1)$ in an inner product space $\mathbb{R}^3$.
4. Check {(1,0),(0,1)} is orthonormal basis or not.
5. Show that the vectors $u = (-2,3,1,4)$ and $v = (1,2,0,-1)$ are orthogonal in R^4.
6. Define inner product.
7. If (x, y) is an inner product on a vector space V, the $2(x, y)$ can be an inner product on V?
8. Find $cos\theta$, where θ is the angle between $A = \begin{pmatrix} 9 & 8 & 7 \\ 6 & 5 & 4 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$, where $< A, B >= tr(B^T A)$.
9. Find the characteristic polynomial of the matrix $A = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 4 & -17 & 8 \end{pmatrix}$.

Part-B

1. Apply the Gram-Schmidt process to transform the basis vector $v_1 = (1,1,1)$, $v_2 = (0,1,1)$ and $v_3 = (0,0,1)$ into an orthogonal basis.
2. Find an orthonormal basis of R^3, given that an arbitrary basis is $\{v_1 = (3,0,4), v_2 = (-1,0,7), v_1 = (2,9,11)\ \}$.
3. Let $V = P(R)$ with the inner product $< f(x), g(x) >= \int_{-1}^{1} f(t)g(t)dt.$ Consider the sub space $P_2(R)$ with standard ordered basis B. Use the Gram-Schmidt process to replace B by an orthogonal basis $\{v_1, v_2, v_3\}$ for $P_2(R)$and use that orthogonal basis to obtain an orthonormal basis for $P_2(R)$.
4. Let $V = C^3$ where C is the set of complex numbers. Define $< x, y >= a_1\overline{b_1} + a_2\overline{b_2} + a_3\overline{b_3}$ where $x = (a_1, a_2, a_3)$ and $y = (b_1, b_2, b_3)$. Verify whether V is an inner product space or not.
5. Let V be a finite dimensional inner product space, and let T be linear operator on V. Then show that there exists a unique linear function $T^*: V \to V$ such that $< T(x), y >=< x, T^*(y) >$ for all $x, y \in V$.
6. Find the orthogonal projection of the vector $u = (-3,-3,8,9)$ on the subspace of R^4 spanned by the vectors $v_1 = (3,1,0,1)$, $v_2 = (1,2,1,1)$ and $v_3 = (-1,0,2,-1)$.
7. Explain the Gram-Schmidt process.
8. In R^4 the set $\{(1,0,1,0), (1,1,1,1), (0,1,2,1)\}$ is linear independent. Using Gram-Schmidt process, obtain an orthonormal set.
9. Let R^3 have the Euclidean inner product. Use Gram-Schmidth process to transform the basis $\{u_1, u_2, u_3\}$ into an othonormal basis, where $u_1 = (1,1,1), u_2 = (0,1,1)\ and\ u_3 = (0,0,1)$.
10. Let S $= \{(1,1,0), (1,-1,1), (-1,1,2)\}$ be an orthogonal set then orthonormal set is $\left\{\frac{1}{\sqrt{2}}(1,1,0), \frac{1}{\sqrt{3}}(1,-1,1),\ \frac{1}{\sqrt{6}}(-1,1,2)\right\}$ both are basis of R^3. Let $x = (2,1,3) \in R^3$. Express x as a linear combination of orthogonal set S and orthonormal set.
11. Consider the vector space $P(t)$ with inner product $< f(x), g(x) >= \int_0^1 f(t)g(t)dt$. By applying the Gram-Schmidt algorithm to the set $\{1, t, t^2\}$, derive the orthogonal set $\{f_0, f_1, f_2\}$.

Unit-4-Matrix Decompositions

Part-A

1. Define a positive definite matrix.
2. Give the conditions to check if a matrix is positive definite.
3. Define singular value decomposition (SVD).

4. What are the singular values?
5. State applications of SVD.

Part-B

1. Find the least square solution of the linear system $Ax = b$ given by $x_1 - x_2 = 4; 3x_1 + 2x_2 = 1; -2x_1 + 4x_2 = 3$.
2. Let $A = M_{mxn}(F)$. Then prove that $rank\ (A^*A) = rank(A)$.
3. Find a singular value decomposition for $A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \end{pmatrix}$.
4. Find a singular value decomposition for $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$.
5. Construct a QR decomposition for the matrix $A = \begin{pmatrix} -4 & 2 & 2 \\ 3 & -3 & 3 \\ 6 & 6 & 0 \end{pmatrix}$.
6. Construct a QR decomposition for the matrix $A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$.
7. Reduce $A = \begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$ to diagonal form by an orthogonal transformation, hence find its nature.