Reg. No. : [ | | | | | | | | | | | | ]

## Question Paper Code : 70141

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Computer Science and Engineering

MA 3354 – DISCRETE MATHEMATICS

(Common to Computer and Communication Engineering/Artificial Intelligence and Data Science/Computer Science and Business Systems/Information Technology)

(Regulations 2021)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Prove that $(p \to (p \vee q))$ is a tautology.

2. Symbolize the statement "All men are mortal".

3. Among 200 people, how many of them were born on the same month?

4. Find the first four terms of the sequence defined by the recurrence relations and initial condition $a_n = a_{n-1}^2$, $a_1 = 2$.

5. Define the pseudograph with an example.

6. Draw the graph of the adjacency matrix $\begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

7. Define monoid.

8. Define the homomorphism of groups.

9. Draw Hasse diagram for $\leq$ relation on $\{0, 2, 5, 10, 11, 15\}$.

10. Define lattice.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $\neg(p \wedge (\neg p \wedge q))$ and $(\neg p \wedge \neg q)$ are logically equivalent without using truth table. (8)

(ii) Using the indirect method, show that $p \to q, q \to r, \neg(p \wedge r)$ $p \vee r \Leftrightarrow r$. (8)

Or

(b) (i) Find a conjunctive normal form of $p \to [(p \to q) \wedge \neg(\neg q \vee \neg p)]$. (8)

(ii) Establish the validity of the following argument : "All integers are rational numbers. Some integers are powers of 2. Therefore, some rational numbers are powers of 2". (8)

12. (a) (i) In a class of 50 students, 20 students play football, and 16 students play hockey. It is found that 10 students play both the games. Find the number of students who play neither. (8)

(ii) Use generating functions to solve the recurrence relation $a_n - 2a_{n-1} - 3a_{n-2} = 0$, $n \geq 2$ with $a_0 = 3$, $a_1 = 1$. (8)

Or

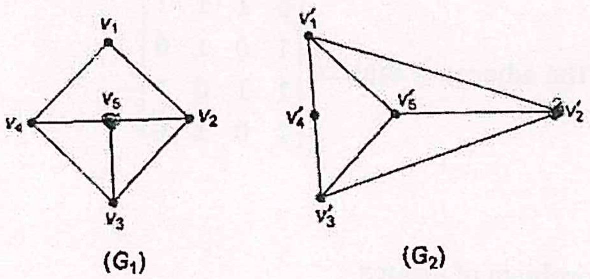
(b) (i) Use mathematical induction to prove that,

$1^2 + 3^2 + 5^2 + \ldots + (2n+1)^2 = \frac{(n+1)(2n+1)(2n+3)}{3}$, whenever $n$ is positive integer. (8)

(ii) Solve the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ for $n \geq 2$, $a_0 = 16$, $a_1 = 80$. (8)

13. (a) (i) Show that the maximum number of edges in a simple graph with '$n$' vertices is $\frac{n(n-1)}{2}$. (8)

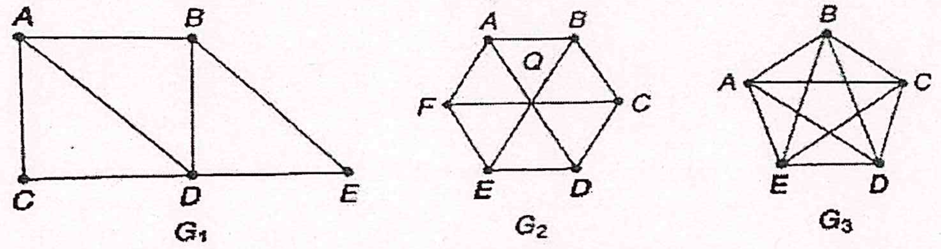
(ii) Establish the isomorphism of the following pairs of graphs, by considering their adjacency matrices. (8)




Or

(b) (i) In any graph $G$, prove that the total number of odd-degree vertices is even. (8)

(ii) Find an Euler path or an Euler circuit, if it exits in each of the following three graphs. If it does not exist, explain why? (8)




14. (a) (i) A subgroup $H$ of a group $G$ is a normal subgroup in $G$ iff each left coset of $H$ in $G$ is equal to the right coset of $H$ in $G$. (8)

(ii) Show that $(Z, *)$ is a group, where $*$ is defined by $a * b = a + b + 1$. (8)

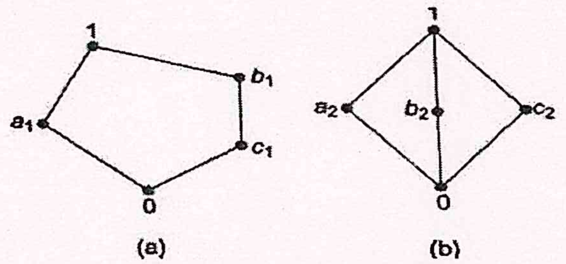
Or

(b) (i) Show that the group $G = \{1, -1, i, -i\}$ is cyclic and find its generators. (6)

(ii) Show that the set $z_4 = \{0, 1, 2, 3\}$ is a commutative ring with respect to the binary operations additive modulo $4(+_4)$ and multiplicative modulo $4(\times_4)$. (10)

15. (a) (i) Show that every chain is a distributive lattice. (8)

(ii) Verify whether the lattices given by the Hasse diagrams in following are distributive. (8)




Or

(b) (i) State and prove De Morgan's law in any Boolean Algebra. (8)

(ii) If $S_n$ is the set of all divisors of the positive integers '$n$' and "aDb" if and only if 'a' divides 'b', prove that $\{S_{24}, D\}$ is a lattice. Find also all the sublattices of $D_{24}$ that contain 5 or more elements. (8)

---

Reg. No. : | | | | | | | | | | | |

**Question Paper Code : 30243**

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third Semester

Computer Science and Engineering

MA 3354 — DISCRETE MATHEMATICS

(Common to : Computer and Communication Engineering/Artificial Intelligence and Data Science/Computer Science and Business Systems/Information Technology)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Construct the truth table for $(\sim p) \vee (\sim q)$.

2. Symbolise the following statement, "All the world loves a lover".

3. How many ways are there to select five players from a 10-member tennis team to make a trip to a match at another school?

4. State the Pigeonhole principle.

5. What is meant by simple graph? Give an example.

6. Draw a graph with the adjacency matrix $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$.

7. What is meant by commutative semi group?

8. Define a field.

9. Draw the Hasse diagram for $(D_{24}, /)$, where $D_{24} = \{1, 2, 3, 4, 6, 8, 12, 24\}$.

10. State De Morgan's law in any Boolean Algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $(\sim p \wedge (\sim q \wedge r)) \vee (q \wedge r) \vee (p \wedge r) \Leftrightarrow r$. (8)

(ii) Show that the following argument is valid, "Every, micro computer has a serial interface port. Some micro computers have a parallel port. Therefore some micro computers have both serial interface port and parallel port". (8)

Or

(b) (i) Find a principal disjunctive normal form and a principal conjunctive normal form of $p \vee (\sim p \rightarrow (q \vee (\sim q \rightarrow r)))$. (8)

(ii) Using the indirect method, Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)Q(x)$. (8)

12. (a) (i) Use mathematical induction to show that $1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}$, whenever $n$ is a positive integer. (8)

(ii) Solve : $T(k) - 7T(k-1) + 10T(k-2) = 6 + 8k$, $T(0) = 1$ $T(1) = 2$. (8)

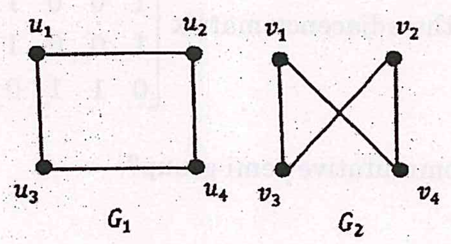
Or

(b) (i) Solve the recurrence relation $y_{n+2} - 4y_{n+1} + 3y_n = 0$, $y_0 = 2$, $y_1 = 4$, using the generating function. (8)

(ii) In a survey of 100 students, it was found that 40 studied Mathematics, 64 studied Physics, 35 studied Chemistry, 1 studied all the three subjects, 25 studied Mathematics and Physics, 3 studied Mathematics and Chemistry, 20 studied Physics and Chemistry. Use the principle of inclusion and exclusion, find the number of students who studied Chemistry only and the number who studied none of these subjects? (8)

13. (a) (i) Prove that the number of vertices of odd degree in a graph $G$ is always even. (8)

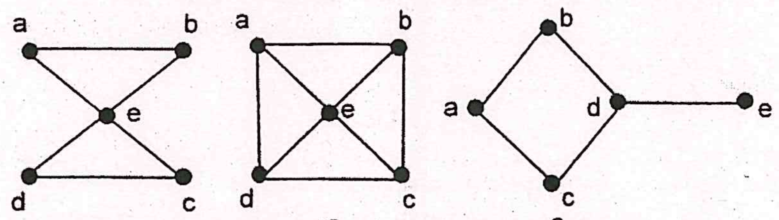
(ii) Determine whether the following pairs of graphs $G_1$ and $G_2$ are isomorphic. (8)




Or

(b) (i) Prove that there is a simple path between every pair of distinct vertices of a connected undirected graph. (8)

(ii) Find an Euler path or an Euler circuit, if it exists in each of the following three graphs. If it does not exist, explain why? (8)




$G_1$ $G_2$ $G_3$

14. (a) (i) Prove that $[Z_5, +_5]$ is an abelian group. (8)

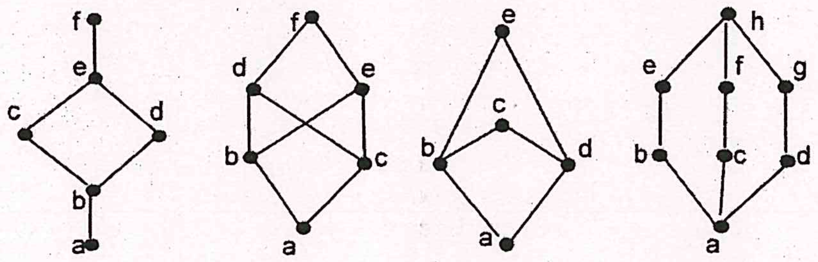
(ii) Show that any two right (left) cosets of H in $G$ are either disjoint or identical. (8)

Or

(b) (i) State and prove Lagrange's Theorem. (8)

(ii) Show that the intersection of two normal subgroups of a group of $(G, *)$ is a normal subgroup of $(G, *)$. (8)

15. (a) (i) Determine whether the following partial ordering sets with the Hasse diagrams are lattices. (8)




(1) (2) (3) (4)

(ii) Let $(L,\leq)$ be a lattice in which $\wedge$ and $\vee$ denote the operations of meet and join respectively. For any $a, b \in L$, prove that $a \leq b \Leftrightarrow a \wedge b = a \Leftrightarrow a \vee b = b$. (8)

Or

(b) (i) If $L$ is a distributive lattice, prove $(a \vee b) \wedge (b \vee c) \wedge (c \vee a) = (a \wedge b) \vee (b \wedge c) \vee (c \wedge a)$. (8)

(ii) In a Boolean algebra, prove

(1) $b \leq c \Rightarrow a \cdot b \leq a \cdot c$ and

(2) $b \leq c \Rightarrow a + b \leq a + c$. (8)

---

Reg. No. : | | | | | | | | | | | | |

**Question Paper Code : 21282**

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Third Semester

Computer Science and Engineering

MA 3354 – DISCRETE MATHEMATICS

(Common to: Computer Science and Engineering (Artificial Intelligence and Machine Learning)/Computer Science and Engineering (Cyber Security)/Computer and Communication Engineering/Artificial Intelligence and Data Science/Computer Science and Business Systems and Information Technology)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the symbolic form of the statement "The automated reply cannot be sent when the file system is full".

2. Check whether the conclusion C : q follows logically from the premises $H_1 : p \to q$ and $H_2 : q$ using truth table technique.

3. How many different bit strings are there of length 9?

4. Determine $n$ if $P(n, 2) = 72$.

5. Define a complete graph.

6. When is a graph called an Eulerian graph?

7. Identify the left cosets of {[0], [3]} in the group $(Z_6, +_6)$.

8. Given $G = \{1, -1, i, -i\}$ be a group and $H = \{-1, 1\}$ be a subgroup of G. Find the index of H in G.

9. Draw the Hasse diagram of $(S_{24}, /)$ where $S_{24}$ is the set of all positive divisors of 24 and / is division.

10. Prove that $(a')' = a$, for all $a \in B$ where B is a Boolean Algebra.

## PART B — (5 × 16 = 80 marks)

11. (a) (i) Construct the truth table for the following statement (8)

$\neg(p \vee (q \wedge r)) \leftrightarrow ((p \vee q) \wedge (p \vee r))$.

(ii) "If there was a ball game, then travelling was difficult. If they arrived on time, then travelling was not difficult. They arrived on time. Therefore, there was no ball game". Show that these statements constitute a valid argument. (8)

Or

(b) (i) Construct the principal disjunctive normal form of $p \rightarrow [(p \rightarrow q) \wedge \neg(\neg q \vee \neg p)]$. (8)

(ii) Use the indirect method to prove that the conclusion $\exists z Q(z)$ from the premises $\forall x(P(x) \rightarrow Q(x))$ and $\exists y P(y)$. (8)

12. (a) (i) Suppose a department consists of eight men and women. In how many ways can we select a committee of (8)

(1) Three men and four women?

(2) Four persons that has at least one woman?

(3) Four persons that has at most one man?

(4) Four persons that has persons of both gender?

(ii) Use generating functions to solve the recurrence relation (8)

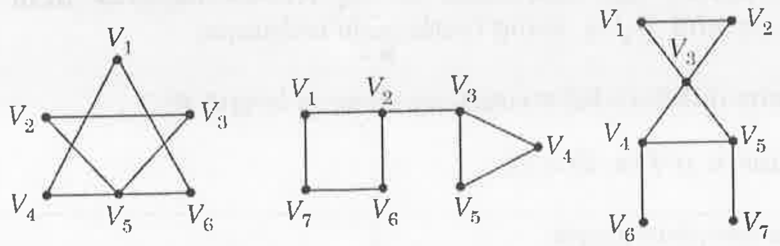
$a_n = 3a_{n-1} + 1;\ n \geq 1$ *with* $a_0 = 1$.

Or

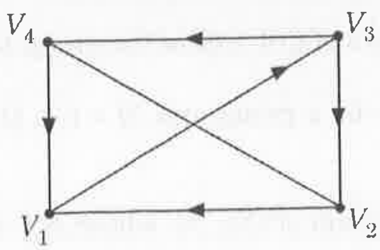
(b) (i) Use mathematical induction to prove that $(3^n + 7^n - 2)$ is divisible by 8, for $n \geq 1$. (8)

(ii) Solve the recurrence relation $a_n = 2a_{n-1} + 2^n, a_0 = 2$. (8)

13. (a) (i) Define a connected graph. Which of the given graphs are connected? (8)



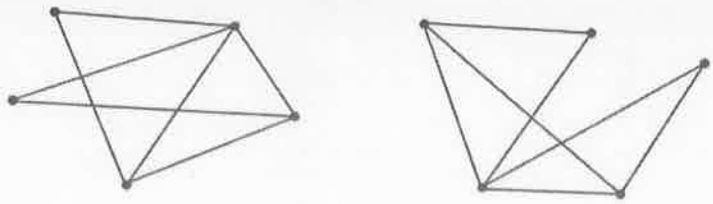

(ii) Explain Euler circuit and Euler path. Determine whether the given graph G has an Euler circuit or Euler path. (8)




Or

(b) (i) Prove that the maximum number of edges in a simple disconnected graph $G$ with '$n$' vertices and '$k$' components is $\frac{(n-k)(n-k+1)}{2}$. (8)

(ii) Examine whether the following pair of graphs are isomorphic. If isomorphic, label the vertices of the two graphs to show that their adjacency matrices are the same. (8)



14. (a) (i) State and prove Lagrange's theorem. (8)

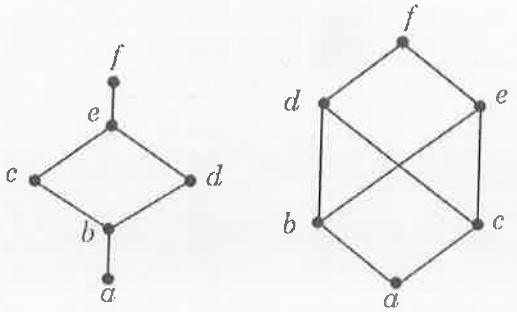
(ii) If $(G, *)$ is a group, prove that $(a*b)^{-1} = b^{-1} * a^{-1}$ for all $a, b \in G$. (8)

Or

(b) Show that $(Z, \oplus, \odot)$ is a commutative ring with identity, where the operations $\oplus$ and $\odot$ are defined, for any $a, b \in Z$ as $a \oplus b = a + b - 1$ and $a \odot b = a + b - ab$. (16)

15. (a) (i) Let $(L, \leq)$ be a lattice and $a, b, c, d \in L$. If $a \leq c$ and $b \leq d$, Then prove that (1) $a \vee b \leq c \vee d$ (2) $a \wedge b \leq c \wedge d$. (8)

(ii) Verify whether the poset represented by the each of the Hasse diagrams are lattices. (8)




Or

(b) (i) State and prove De Morgan's laws in any Boolean Algebra. (8)

(ii) If $a, b \in S = \{1, 2, 4, 8, 16\}$ and $a \vee b = l.c.m.$ of $\{a, b\}$, $a \wedge b$ = $g.c.d$ of $\{a, b\}$ and $a' = 16/a$, then show that $\{S, \vee, \wedge, ', 1, 16\}$ is not a Boolean algebra. (8)

———————

Reg. No. :

# Question Paper Code : 51325

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Third Semester

Computer Science and Engineering

MA 3354 – DISCRETE MATHEMATICS

(Common to : Computer Science and Engineering (Artificial Intelligence and Machine Learning)/Computer Science and Engineering (Cyber Security)/ Computer and Communication Engineering/Artificial Intelligence and Data Science/ Computer Science and Business Systems/Information Technology)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Show that $p \to q$ and $\neg p \vee q$ are equivalent.

2. Translate the statement 'Every real number expect zero has a multiplicative inverse' into the statement involving nested quantifiers.

3. The chairs of an auditorium are to be labeled with an uppercase English letter followed by a positive integer not exceeding 100. What is the largest number of chairs that can be labeled differently?

4. State the pigeonhole principle.

5. Define degree of a vertex in an undirected graph.

6. Define path.

7. Is the set of integers under ordinary multiplication a group?

8. Prove that for each element $a$ in a group $G$, there is a unique element $b$ in $G$ such that $ab = ba = e$.

9. Show that the 'greater than or equal' relation ($\geq$) is a partial ordering on the set of integers.

10. Is the poset ($\mathbb{Z}^+$, $|$) a lattice?

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) Find the principle disjunctive normal form of $(P \to Q) \wedge (P \rightleftarrows R)$. (8)

(ii) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. (8)

Or

(b) (i) Prove that if $n = ab$, where $a$ and $b$ are positive integers, then $a \leq \sqrt{n}$ or $b \leq \sqrt{n}$. (8)

(ii) Show that $(\forall x)[P(x) \vee Q(x)] \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$. (8)

12. (a) (i) Show that if $n$ is an integer greater than 1, then $n$ can be written as the product of primes. (8)

(ii) During a month with 30 days, a baseball team plays at least one game a day, but no more than 45 games. Show that there must be a period of some number of consecutive days during which the team must play exactly 14 games. (8)

Or

(b) (i) Find the number of 2-permutations with unlimited repetitions of $\{a, b, c, d\}$. (8)

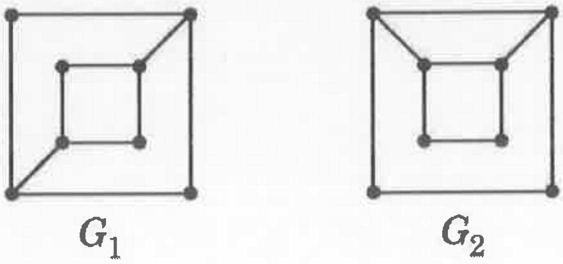
(ii) Solve $s(n+2) - 5s(n+1) + 6s(n) = 0$ for $n \geq 0$ with $s(0) = 1$, $s(1) = 1$. (8)

13. (a) (i) Prove that a simple connected graph is bipartite if and only if it contains no odd cycle. (8)

(ii) Draw the graph with the adjacency matrix $\begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 0 \end{bmatrix}$ with respect to the ordering of $a, b, c, d$. (8)

Or

(b) (i) Check whether the graphs are isomorphic or not? (8)




(ii) Prove that, if $G$ is a simple connected graph with $n \geq s$ vertices and $d(u) \geq \frac{n}{2}$, for all $u \in V(G)$, then $G$ is Hamiltonian graph. (8)

14. (a) (i) Prove that the subgroup of a cyclic group must be cyclic subgroup. (8)

(ii) Prove that a subgroup $H$ of $G$ is normal in $G$ if and only if $xHx^{-1} \subseteq H$ for all $x$ in $G$. (8)

Or

(b) (i) State and prove Lagrange's theorem. (8)

(ii) Prove that the Kernal of a homomorphism $f$ from the group $\langle G, * \rangle$ to a group $\langle H, \Delta \rangle$ is a normal subgroup of the group $\langle G, * \rangle$. (8)

15. (a) (i) Draw the Hasse diagram representing the partial ordering $\{(a, b) \mid a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 8, 12\}$. (8)

(ii) Prove that every chain is a distributive lattice. (8)

Or

(b) (i) Find the values of the Boolean function represented by $F(x, y, z) = xy + \bar{z}$. (8)

(ii) Prove that the De Margon laws are valid in a Boolean Algebra. (8)