Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 51326

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

Third/Fourth Semester

Biomedical Engineering

MA 3355 — RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to : Electronics and Communication Engineering/Electronics and Telecommunication Engineering/Medical Electronics)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

(Statistical tables may be permitted)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. State the axioms of probability.

2. The mean of Binomial distribution is 20 and standard deviation is 4. Find the parameters of this distribution.

3. State central limit theorem.

4. If X and Y are independent random variables prove that $\operatorname{Cov}(X, Y) = 0$.

5. State the four classifications of Random processes.

6. State Chapman Kolmogorov equations.

7. Is the set of all matrices A such that $\det(A) = 0$ subspaces of the matrix M_{mn}? Justify.

8. Why $v_1 = (-1, 2, 4)$ and $v_2 = (5, -10, -20)$ in R^3 are linearly dependent? Explain it.

9. What is Identity transformation?

10. Write eigen values of the matrices A^2 and A^{-1} given that matrix $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) The CDF of the random variable X is defined by

$$F_X(x) = \begin{cases} 0, & x < 2 \\ C(x-2), & 2 \le x < 6 \\ 1, & x \ge 6 \end{cases}$$

(1) What is the value of C? (8)

(2) With the above value of C, what is $P[X > 4]$?

(3) With the above value of C, what is $P[3 \le X \le 5]$?

(ii) The average percentage of marks of candidates in an examination is 42 will a standard deviation of 10, the minimum for a pass is 50%. If 1000 candidates appear for the examination, how much can be expected marks. If it is required, that double that number should pass, what should be the average percentage of marks? (8)

Or

(b) (i) A shopping cart contains ten books whose weights are as follows:

There are four books with a weight of 1.8 lbs each, one book with a weight of 2 lbs, two books with a weight of 2.5 lbs each, and three books with a weight of 3.2 lbs each.

(1) What is the mean weight of the books?

(2) What is the variance of the weights of the books? (8)

(ii) Given that X is normally distribution with mean 10 and probability $P(X > 12) = 0.1587$. What is the probability that X will fall in the interval $(9, 11)$. (8)

12. (a) The joint PDF of the random variables X and Y is defined as follows :

$$f_{XY}(x, y) = \begin{cases} 25e^{-5y}, & 0 < x < 0.2,\ y > 0 \\ 0 & ,\ \text{otherwise} \end{cases}$$

(i) Find the marginal PDFs of X and Y

(ii) What is the correlation coefficient of X and Y? (16)

Or

(b) (i) The details pertaining to the experience of technicians in a company (in a number of years) and their performance rating is provided in the table below. Using these values, fit the straight line. Also estimate the performance rating for a technician with 20 years of experience. (10)

Experience of Technicians (in year)	16	12	18	4	3	10	5	12
Performance rating	87	88	89	68	78	80	75	83

(ii) The joint PDF of two random variables X and Y is given by $f_{XY}(x, y)$. If we define the random variable $U = XY$, determine the PDF of U. (6)

13. (a) (i) Suppose we are interested in $X(t)$ but we can observe only $Y(t) = X(t) + N(t)$ where $N(t)$ is a noise process that interferes with our observation of $X(t)$. Assume $X(t)$ and $N(t)$ are independent wide sense stationary processes with $E[X(t)] = \mu_X$ and $E[N(t)] = \mu_N = 0$. Is $Y(t)$ wide sense stationary? (8)

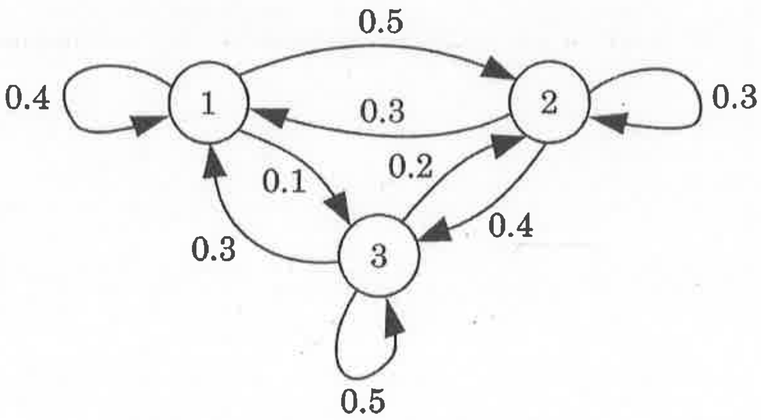
(ii) Cars, trucks, and buses arrive at a toll booth as independent Poisson processes with rates $\lambda_c = 1.2$ cars/minute, $\lambda_t = 0.9$ trucks/minute, and $\lambda_b = 0.7$ buses/minute. In a 10-minute interval, what is the PMF of N, the number of vehicles (cars, trucks, or buses) that arrive? (8)

Or

(b) (i) Check whether the Markov chain with transition probability matrix

$$p = \begin{bmatrix} 0 & 1 & 0 \\ \frac{1}{2} & 0 & \frac{1}{2} \\ 0 & 1 & 0 \end{bmatrix}$$ is irreducible or not? (8)

(ii) Find the transition probability matrix and the limiting-state probabilities of the process represented by the state-transition diagram shown in Figure (8)



14. (a) (i) Check whether the set of all pairs of real numbers of the form $(1, x)$ with the operations $(1, x_1) + (1, x_2) = (1, x_1 + x_2)$ and $k(1, x) = (1, kx)$ is vector space. (8)

(ii) Consider the vectors $u = (1, 2, -1)$ and $v = (6, 4, 2)$ in R^3. Show that $w = (9, 2, 7)$ is a linear combination of u and v. (8)

Or

(b) (i) Show that the vectors $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$, $v_3 = (3, 3, 4)$ form a basis for R^3. (8)

(ii) Find the dimension and a basis for the solution space W of the system of homogeneous equation given below. (8)

$$x_1 + 2x_2 + 2x_3 - x_4 + 3x_5 = 0$$
$$x_1 + 2x_2 + 3x_3 + x_4 + x_5 = 0$$
$$3x_1 + 6x_2 + 8x_3 + x_4 + 5x_5 = 0$$

15. (a) (i) State and prove Dimension theorem. (10)

(ii) Show that the transformation $T : R^3 \to R^2$ defined by $T(x, y, z) = (z, x + y)$ is linear. (6)

Or

(b) (i) Find an orthonormal basis of R^3, given that an arbitrary basis is $\{v_1 = (3, 0, 4), v_2 = (-1, 0, 7), v_3 = (2, 9, 11)\}$. (8)

(ii) Find a basis and dimension of R_T and N_T for the linear transformation $T : R^3 \to R^3$ defined by

$T(x_1, x_2, x_3) = (x_1 - x_2 + 2x_3, 2x_1 + x_2, -x_1 - 2x_2 + 2x_3)$. (8)

Reg. No. :

Question Paper Code : 21283

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

Third/Fourth Semester

Biomedical Engineering

MA 3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to : Electronics and Communication Engineering/ Electronics and Telecommunication Engineering/ Medical Electronics)

(Regulations – 2021)

Time : Three hours Maximum : 100 marks

(Codes/ Tables/ Charts to be permitted, if any may be indicated: Normal table to be permitted.)

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If A, B and C are any 3 events such that $P(A) = P(B) = P(C) = 1/4$, $P(A \cap B) = P(B \cap C) = 0$, $P(A \cap C) = 1/8$. Find the probability that atleast 1 of the events A, B and C occurs.

2. A continuous random variable X that can assume any value between $x = 2$ and $x = 5$ has a density function given by $f(x) = (2/27)(1+x)$. Find $P(X < 4)$.

3. State the properties of the distribution function of a two dimensional random variable (X, Y).

4. The life time of a certain brand of an electric bulb may be considered a random variable with mean 1200h and standard deviation 250h. Find the probability, using central limit theorem, that the average lifetime of 60 bulbs exceeds 1250h.

5. A radioactive source emits particles at a rate of 5 per minute in accordance with Poisson process. Each particle emitted has probability 0.6 of being recorded. Find the probability that 10 particles are recorded in 4-minute period.

6. State the discrete random sequence. Give an example.

7. Determine whether the vectors $v_1 = (1, -2, 3)$, $v_2 = (5, 6, -1)$ and $v_3 = (3, 2, 1)$ form a linearly dependent or a linearly independent set.

8. Does a line passing through origin of $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$?

9. State the dimension theorem in linear algebra.

10. Find the angle between two vectors $u = (4, 3, 1, -2)$ and $v = (-2, 1, 2, 3)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) For a certain binary communication channel, the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that a '0' is transmitted is 0.4, find the probability that (1) a '1' is received and (2) a '1' was transmitted given that '1' was received. (8)

(ii) If X represents the outcome, when a fair dice is tossed, find the moment generating function of X and hence find $E(X)$ and $Var(X)$. (8)

Or

(b) (i) An irregular 6-faced dice is such that the probability that it gives 3 even numbers in 5 throws is twice the probability that it gives 2 even numbers in 5 throws. How many sets of exactly 5 trials can be expected to give no even number out of 2500 sets? (8)

(ii) In an engineering examination, a student is considered to have failed, secured second class, first class and distinction, according as he scores less than 45% between 45% and 60%, between 60% and 72% and above 75% respectively. In particular year 10% of the students failed in the examination and 5% of the students got distinction. Find the percentages of students who have got first class and second class. (Assume normal distribution of marks). (8)

12. (a) (i) The joint probability mass function of (X, Y) is given by $p(x, y) = k(2x + 3y)$, $x = 0,1,2$; $y = 1,2,3$. Find all the marginal probability distributions. (8)

(ii) The joint probability density function of a two-dimensional random variable (X, Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}$, *if* $0 \le x \le 2, 0 \le y \le 1$.

Compute $P(X > 1)$, $P(Y < \frac{1}{2})$, and $P(X > 1 / Y < \frac{1}{2})$ (8)

Or

(b) If X and Y each follow an exponential distribution with parameter 1 and are independent, find the pdf of $U = X - Y$. (16)

13. (a) A man either drives a car or catches a train to go to office each day. He never goes 2 days in a row by train but if he drives one day, then the next day he is just as likely to drive again as he is to travel by train. Now suppose that on the first day of the week, the man tossed a fair dice and drove to work if and only if a 6 appeared. Find

(i) the probability that he takes a train on the third day

(ii) the probability that he drives to work in the long run. (16)

Or

(b) The transition probability matrix of a Markov chain $\{X_n\}$, n=1,2,3... having 3 states 1, 2, and 3 is $P = \begin{pmatrix} 0.1 & 0.5 & 0.4 \\ 0.6 & 0.2 & 0.2 \\ 0.3 & 0.4 & 0.3 \end{pmatrix}$ and the initial distribution is $p^{(0)} = (0.7, 0.2, 0.1)$. Find

(i) $P(X_2 = 3)$ and

(ii) $P(X_3 = 2, X_2 = 3, X_1 = 3, X_0 = 2)$. (16)

14. (a) Show that the set V of all 2×2 matrices with real entries is a vector space if addition is defined to be matrix addition and scalar multiplication is defined to be matrix scalar multiplication. (16)

Or

(b) (i) Find a basis for the space spanned by the vectors $v_1 = (1, -2, 0, 0, 3)$, $v_2 = (2, -5, -3, -2, 6)$, $v_3 = (0, 5, 15, 10, 0)$ and $v_4 = (2, 6, 18, 8, 6)$. (8)

(ii) Let $v_1 = (1, 2, 1)$, $v_2 = (2, 9, 0)$ and $v_3 = (3, 3, 4)$. Show that the set $S = \{v_1, v_2, v_3\}$ is basis for $\mathbb{R}^3$. (8)

15. (a) (i) Find the basis for the nullspace of $\begin{pmatrix} 2 & 2 & -1 & 0 & 1 \\ -1 & -1 & 2 & -3 & 1 \\ 1 & 1 & -2 & 0 & -1 \\ 0 & 0 & 1 & 1 & 1 \end{pmatrix}$. (8)

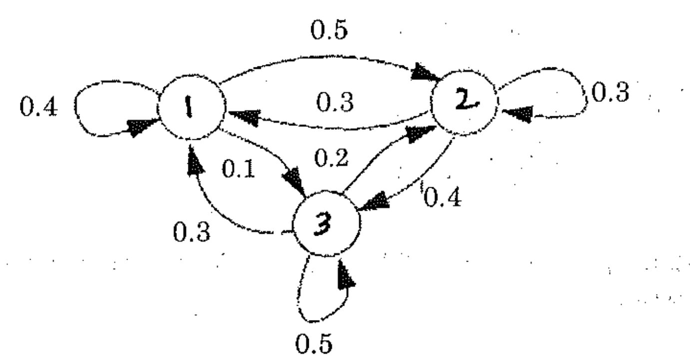
(ii) Let $T : \mathbb{R}^2 \to \mathbb{R}^3$ be the linear transformation defined by $T\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x_2 \\ -5x_1 + 13x_2 \\ -7x_1 + 16x_2 \end{pmatrix}$. Find the matrix for the transformation T with respect to the basis $B = \{u_1, u_2\}$ for $\mathbb{R}^2$ and $B' = \{v_1, v_2, v_3\}$ for $\mathbb{R}^3$, where $u_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$, $u_2 = \begin{pmatrix} 5 \\ 2 \end{pmatrix}$, $v_1 = \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$, $v_2 = \begin{pmatrix} -1 \\ 2 \\ 2 \end{pmatrix}$, $v_3 = \begin{pmatrix} 0 \\ 1 \\ 2 \end{pmatrix}$. (8)

Or

(b) (i) Apply the Gram-Schmidt process to transform the basis vector $u_1 = (1,1,1)$, $u_2 = (0,1,1)$ and $u_3 = (0,0,1)$ into an orthonormal basis. (8)

(ii) Find the least squares solution of the linear system $Ax = b$ given by $x_1 - x_2 = 4$; $3x_1 + 2x_2 = 1$; $-2x_1 + 4x_2 = 3$. (8)

(b) Find the transition probability matrix and the limiting-state probabilities of the process represented by the state-transition diagram shown in the below diagram. (16)



14. (a) Determine whether the set of all pairs of real numbers of the form (1, x) with the operations (1, y)+(1, y') = (1, y + y') and k(1, y) = (1, ky) is a vector space or not. If not, identify the vector space axioms that fail to hold. (16)

Or

(b) Find a basis and the dimension of the solution space of the homogeneous system $x_1+3x_2-2x_3+2x_5=0$; $2x_1+6x_2-5x_3-2x_4+4x_5-3x_6=0$; $5x_3+10x_4+15x_6=0$; $2x_1+6x_2+8x_4+4x_5+18x_6=0$. (16)

15. (a) Assume that the vector space R^3 has the Euclidean inner product. Apply the Gram-Schmidt process to transform the basis vectors $u_1=(1, 1, 1)$, $u_2=(0, 1, 1)$, $u_3=(0, 0, 1)$ into an orthogonal basis $\{v_1, v_2, v_3\}$, and then normalize the orthogonal basis vectors to obtain an orthonormal basis $\{q_1, q_2, q_3\}$. (16)

Or

(b) Find the least squares solution, the least squares error vector, and the least squares error of the linear system (16)

$x_1-x_2=4$

$3x_1+2x_2=1$

$-2x_1+4x_2=3$

Reg. No. :

Question Paper Code : 30244

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

Third/Fourth Semester

Bio Medical Engineering

MA 3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to : Electronics and Communication Engineering/Electronics and Telecommunication Engineering/Medical Electronics)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Statistical Tables should be provided

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Two fair dice are tossed. Find the probability of the outcome of the second die is greater than the outcome of the first die.

2. A bag contains eight red balls, four green balls, and eight yellow balls. A ball is drawn at random from the bag, and it is not a red ball. What is the probability that it is a green ball?

3. Given two random variables X and Y with the joint CDF $F_{XY}(x,y)$ and marginal CDFs $F_x(x)$ and $F_y(y)$, respectively, compute the joint probability that X is greater than a and Y is greater than b.

4. The joint PMF of two random variables X and Y is given by $P_{XY}(x,y)=\begin{cases}\frac{1}{18}(2x+y), & x=1,2;\ y=1,2\\ 0, & otherwise\end{cases}$. What is the marginal PMF of X?

5. Customers arrive at a grocery store in a Poisson manner at an average rate of 10 customers per hour. The amount of money that each customer spends is uniformly distributed between \$8.00 and \$20.00. What is the average total amount of money that customers who arrive over a two-hour interval spend in the store?

6. What are the four basic types of Markov processes?

7. Consider the vectors $u=(1,2,-1)$ and $v=(6,4,2)$ in R^3. Show that $w=(9,2,7)$ is a linear combination of u and v.

8. Determine whether the vectors $v_1=(1,-2,3), v_2=(5,6,-1), v_3=(3,2,1)$ are linearly independent or linearly dependent in R^3.

9. Let $T:R^3 \to R^3$ be the orthogonal projection onto the xy-plane. Find the range and kernel of the transformation.

10. Prove that the vectors $u=(1,1)$ and $v=(1,-1)$ are orthogonal with respect to the Euclidean inner product on R^2.

PART B — (5 × 16 = 80 marks)

11. (a) (i) A student buys 1000 integrated circuits (ICs) from supplier A, 2000 ICs from supplier B, and 3000 ICs from supplier C. He tested the ICs and found that the conditional probability of an IC being defective depends on the supplier from whom it was bought. Specifically, given that an IC came from supplier A, the probability that it is defective is 0.05; given that an IC came from supplier B, the probability that it is defective is 0.10; and given that an IC came from supplier C, the probability that it is defective is 0.10. If the ICs from the three suppliers are mixed together and one is selected at random, what is the probability that it is defective? (8)

(ii) State and prove the "forgetfulness" Property of the Geometric Distribution. (8)

Or

(b) (i) The weights in pounds of parcels arriving at a package delivery company's warehouse can be modeled by an $N(5;16)$ normal random variable, X. (8)

(1) What is the probability that a randomly selected parcel weighs between 1 and 10 pounds?

(2) What is the probability that a randomly selected parcel weighs more than 9 pounds?

(ii) The lengths of phone calls at a certain phone booth are exponentially distributed with a mean of 4 minutes. I arrived at the booth while Tom was using the phone, and I was told that he had already spent 2 minutes on the call before I arrived.

(1) What is the average time I will wait until he ends his call?

(2) What is the probability that Tom's call with last between 3 minutes and 6 minutes after my arrival? (8)

12. (a) The joint CDF of two discrete random variables X and Y is given as follows: (16)

$$F_{xy}(x,y)=\begin{cases}\frac{1}{8}, x=1, y=1\\ \frac{5}{8}, x=1, y=2\\ \frac{1}{4}, x=2, y=1\\ 1, x=2, y=2\end{cases}$$

Determine the joint PMF of X and Y; Marginal PMF of X and Marginal PMF of Y.

Or

(b) The joint PDF of the random variables X and Y is defined as follows:

$$f_{x,y}(x,y)=\begin{cases}25e^{-5y}, & 0<x<0.2,\ y>0\\ 0, & elsewhere\end{cases}$$

What is the covariance of X and Y? (16)

13. (a) A company cafeteria opens daily on weekdays at 8 a.m. Studies indicate that the employees arrive at the cafeteria over its normal business hours in a Poisson manner. However, the arrival rate varies with the time of the day. In particular, the following observation has been made:

(i) During the first three hours from when the cafeteria opens for business, there is a steady increase in the customer arrival rate from 4 per hour to 16 per hour.

(ii) Then the arrival rate remains constant at 16 customers per hour for the next two hours.

(iii) Finally the arrival rate uniformly declines to 0 per hour in the next 2 hours.

(1) What is the probability that no employee arrives at the cafeteria during the first two hours?

(2) What is the expected number of arrivals during the first four hours? (16)

Or

Reg. No. :

Question Paper Code : 70142

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

Third Semester

Electronics and Telecommunication Engineering

MA 3355 – RANDOM PROCESSES AND LINEAR ALGEBRA

(Common to: B.E. Electronics and Communication Engineering)

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Using the axioms of probability, prove $P(A^c) = 1 - P(A)$.

2. Consider a random experiment of tossing a fair coin three times. If X denotes the number heads obtained find, $P(X < 2)$.

3. For a bi-variate random variable (XY), prove that if X and Y are independent, then every event $a < X \le b$ is independent of the other event $c < X \le d$.

4. Let the joint probability mass function of (X, Y) be given by $P_{xy}(x, y) = \begin{cases} k(x+y) & x = 1,2,3; y = 1.2 \\ 0, & otherwise \end{cases}$. Find the value of k.

5. Let $X_1, X_2 \ldots$ be independent Bernoulli random variables with $P(X_n = 1) = p$ and $P(X_n = 0) = q$ for all n. Describe the Bernoulli process.

6. Consider a Markov chain with two states and transition probability matrix $P = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$. Find the stationary distribution of the chain.

7. Determine whether the vectors $u = (1,1,2), v = (1,0,1)$, and $w = (2,1,3)$ span the vector space R^3.

8. Is a set of all vectors of the form $(a,1,1)$, where a is real, a subspace of R^3? Justify.

9. Find the kernel and range of the identity operator.

10. Show that the vectors $u = (-2, 3, 1, 4)$ and $v = (1, 2, 0, -1)$ are orthogonal in R^4.

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) A lot of 100 semiconductor chips contain 20 that are defective. Two are selected randomly, without replacement, from the lot.

(1) What is the probability that the first one selected is defective?

(2) What is the probability that the second one selected is defective given that the first one was defective?

(3) What is the probability that both are defective? (8)

(ii) A company producing electric relays has three manufacturing plants producing 50, 30, and 20 percent respectively of its product. Suppose that the probabilities that a relay manufactured by these plants is defective are 0.02, 0.05 and 0.01 respectively. If a relay selected at random is found to be defective, what is the probability that it was manufactured by plant 2? (8)

Or

(b) (i) All manufactured devices and machines fail to work sooner or later. Suppose that the failure rate is constant and the time to failure (in hours) is an exponential random variable X with parameter λ. Measurements shows that the probability that the time to failure for computer memory chips in a given class exceeds 10^4 hours is e^{-1}. Find the value of λ and calculate the time X_0 such that the probability that the time to failure is less than X_0 is 0.05. (8)

(ii) A production line manufactures 1000 ohm resistors that have 10% tolerance. Let X denotes the resistance of a resistor. Assuming that X is a normal random variable with mean 1000 and variance 2500, find the probability that a resistor picked at random will be rejected. (8)

12. (a) Consider an experiment of drawing randomly three balls from an urn containing two red, three white, and four blue balls. Let (X, Y) be a bi-variate random variable where X and Y denote respectively the number of red and white balls chosen.

(i) Find the range of (X, Y).

(ii) Find the joint probability mass function of (X,Y).

(iii) Find the marginal probability function of X and Y.

(iv) Are X and Y independent? (16)

Or

14. (a) Determine whether the set of all pairs of real numbers (x, y) with the operations $(x, y)+(p, q)=(x+p+1, y+q+1)$ and $k(x, y)=(kx, ky)$ is a vector space or not. If not, list all the axioms that fail to hold. (16)

Or

(b) Determine the basis and the dimension of the homogeneous system $2x_1+2x_2-x_3+x_5=0; -x_1-x_2+2x_3-3x_4+x_5=0;$ $x_1+x_2-2x_3-x_5=0 \;\; x_3+x_4+x_5=0$. (16)

15. (a) (i) State and prove the dimension theorem for linear transformation. (8)

(ii) Let $T: R^2 \to R^3$ be a linear transformation defined by $T\begin{pmatrix} x \\ y \end{pmatrix}=\begin{bmatrix} y \\ -5x+13y \\ -7x+16y \end{bmatrix}$. Find the matrix for the transformation T with respect to the bases $B=\{u_1, u_2\}$ for R^2 and $B_1=\{v_1, v_2, v_3\}$ for R^3 where $u_1=\begin{bmatrix} 3 \\ 1 \end{bmatrix}, u_2=\begin{bmatrix} 5 \\ 2 \end{bmatrix}, v_1=\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}, v_2=\begin{bmatrix} -1 \\ 2 \\ 2 \end{bmatrix}, v_3=\begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$. (8)

Or

(b) Find the orthogonal projection of the vector $u=(-3,-3,8,9)$ on the subspace of R^4 spanned by the vectors $v_1=(3,1,0,1), v_2=(1,2,1,1)$, $v_3=(-1,0,2,-1)$. (16)