Reg. No. :

Question Paper Code : 41520

B.E./B.Tech. DEGREE EXAMINATIONS, JANUARY 2022.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to All Branches (Except : Marine Engineering)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If 2, −1, −3 are the eigenvalues of a matrix "A", then find the eigenvalues of the matrix $A^2 - 2I$.

2. Write down the matrix for the following quadratic form:

   $2x_1^2 - 2x_2^2 + 4x_3^2 + 2x_1 x_2 - 6x_1 x_3 + 6x_2 x_3$.

3. Find the domain of the function $f(x) = \dfrac{2x^3 - 5}{x^2 + x - 6}$.

4. Evaluate the limit $\lim_{x \to 1} \dfrac{x^2 - 4x}{x^2 - 3x - 4}$.

5. If $u = x^3 + y^3$ where $x = a\cos t$ and $y = b\sin t$ then find $\dfrac{du}{dt}$.

6. If $u = \dfrac{2x - y}{2}$ and $v = \dfrac{y}{z}$ then find $\dfrac{\partial(u, v)}{\partial(x, y)}$.

7. Given that $\int_0^{10} f(x)\, dx = 17$ and $\int_0^{8} f(x)\, dx = 12$ then find $\int_8^{10} f(x)\, dx$.

8. Determine whether the integral $\int_0^{\infty} \frac{dx}{x^2+4}$ is convergent or divergent.

9. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \int_0^{2\cos\theta} d\theta\ dr$.

10. Evaluate $\int_0^1 \int_0^2 \int_0^3 \left[x\ y^2\ z\right] dx\ dy\ dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $A=\begin{bmatrix} 2 & 2 & 0 \\ 2 & 1 & 1 \\ -7 & 2 & -3 \end{bmatrix}$. (8)

(ii) Using Cayley — Hamilton theorem find the inverse of the matrix $A=\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $3x_1^2 + 3x_2^2 + 3x_3^2 + 2x_1\ x_2 + 2x_1\ x_3 - 2x_2\ x_3$ to canonical form through an orthogonal transformation. Also find its nature, rank, index and signature. (16)

12. (a) (i) If $x^2+y^2=25$, then find $\frac{dy}{dx}$ and also find an equation of the tangent line to the curve $x^2+y^2=25$ at the point $(3,4)$. (8)

(ii) If $f(x)=xe^x$ then find $f'(x)$. Also find the n-th derivative $f^n(x)$. (8)

Or

(b) (i) Differentiate the function $f(x)=\frac{\sec x}{1+\tan x}$. For what values of x, the graph of $f(x)$ has a horizontal tangent? (8)

(ii) Find the absolute maximum and absolute minimum values of the function $f(x)=3x^4-4x^3-12x^2+1$ on the interval $[-2, 3]$. (8)

13. (a) (i) If $u = \log[\tan x + \tan y + \tan z]$ then find the value of $\sin 2x \frac{\partial u}{\partial x} + \sin 2y \frac{\partial u}{\partial y} + \sin 2z \frac{\partial u}{\partial z}$. (8)

(ii) Find the minimum value of $f(x, y) = x^2 + y^2 + 6x + 12$. (8)

Or

(b) (i) Expand $f(x, y) = e^x \sin y$ in terms of powers of "x" and "y" up to third degree terms by using Taylor's series. (8)

(ii) Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. (8)

14. (a) (i) Evaluate $\int \cos^n x \, dx$ by using integration by parts. (8)

(ii) Evaluate $\int \frac{dx}{\sqrt{3x - x^2 - 2}}$. (8)

Or

(b) (i) Evaluate $\int \frac{x^2 + 2x - 1}{2x^3 + 3x^2 - 2x} dx$ by using the method of partial fractions. (8)

(ii) Evaluate $\int \frac{2x + 3}{x^2 + x + 1} dx$. (8)

15. (a) (i) Evaluate $\iint [x\,y]\, dx\, dy$ where the region of integration is bounded by the lines x-axis, $x = 2a$ and the curve $x^2 = 4ay$. (8)

(ii) Change the order of the integration in $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} [x\,y]\, dy\, dx$ and hence evaluate it. (8)

Or

(b) (i) Evaluate $\int_0^a \int_y^a \left[\frac{x}{x^2 + y^2}\right] dx\, dy$ by changing into polar coordinates. (8)

(ii) Evaluate $\int_0^{2a} \int_0^x \int_y^x [x\,y\,z]\, dz\, dy\, dx$. (8)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 30515

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022.

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to: All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Two eigen values of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 0. Find the third eigen value and also find the product of eigen values of A.

2. Write the quadratic form corresponding to the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -2 & -4 \\ 3 & -4 & -3 \end{bmatrix}$.

3. Find the domain of the function $f(x) = \frac{1}{x^2 - x}$.

4. Prove that $\lim_{x \to 0} \frac{|x|}{x}$ does not exist.

5. Find $\frac{dy}{dx}$, if $x^3 + y^3 = 3axy$.

6. If $u = \frac{y^2}{x}$ and $v = \frac{x^2}{y}$, then find the Jacobian $\frac{\partial(u,v)}{\partial(x,y)}$.

7. Evaluate $\int \frac{\cos\theta}{\sin^3\theta} d\theta$ by the method of substitution.

8. Determine the following integral is convergent or divergent. $\int_0^{\infty} e^x dx$.

9. Evaluate $\int_1^2\int_2^5 [xy]dxdy$.

10. Find the limits of the integration $\iint_R f(x,y)dxdy$ where R is the region bounded by the lines $x=0$, $y=0$ and $x+y=2$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors for the matrix $A=\begin{bmatrix}1 & 0 & -1\\ 1 & 2 & 1\\ 2 & 2 & 3\end{bmatrix}$ (8)

(ii) Using Cayley-Hamilton theorem, find the inverse of the matrix $A=\begin{bmatrix}4 & 6 & 6\\ 1 & 3 & 2\\ -1 & -4 & -3\end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $8x_1^2+7x_2^2+3x_3^2-12x_1x_2-8x_2x_3+4x_3x_1$ to the canonical form through an orthogonal transformation. (16)

12. (a) (i) Find the equation of the tangent line to the curve $y=\frac{e^x}{(1+x^2)}$ at the point $(1, e/2)$. (8)

(ii) Find the absolute maximum and minimum values of the function $f(x)=\log(x^2+x+1)$ in $[-1, 1]$. (8)

Or

(b) (i) Show that the function $f(x)$ is continuous on $(-\infty,\infty)$ $f(x)=\begin{cases}1-x^2; & x\le 1\\ \log x; & x\ge 1\end{cases}$ (8)

(ii) Find the local maxima and minima for the function of the curve $y=x^4-4x^3$. (8)

13. (a) (i) If $u = \sin^{-1}\left[\frac{x^3 - y^3}{x + y}\right]$ then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$. (8)

(ii) Find the maximum and minimum values of $f(x,y) = x^2 - xy + y^2 - 2x + y$. (8)

Or

(b) (i) Using Taylor's series, expand $f(x,y) = x^2y + \sin y + e^x$ upto the second degree terms at the point $(1, \pi)$. (8)

(ii) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box requiring the least material for its construction. (8)

14. (a) (i) Evaluate $\int x^2 e^x dx$ by using integration by parts. (8)

(ii) Evaluate $\int \frac{dx}{\sqrt{3x^2 + x - 2}}$ (8)

Or

(b) (i) Evaluate $\int \frac{x + 4}{6x - 7 - x^2} dx$. (8)

(ii) Evaluate $\int_{-\pi/4}^{\pi/4} [\tan^2 x \sec^2 x] dx$. (8)

15. (a) (i) Change the order of integration in $\int_0^a \int_x^a (x^2 + y^2) dy dx$ and hence evaluate it. (8)

(ii) Evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$ by changing into polar coordinates. (8)

Or

(b) (i) Evaluate $\iint (x^2y + xy^2)\, dx dy$ over the area between $y = x^2$ and $y = x$. (8)

(ii) Evaluate $\int_0^1 \int_0^x \int_0^{\sqrt{x+y}} [z]\, dz\, dy\, dx$. (8)

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Reg. No. : E N G G T R E E . C O M

Question Paper Code : 60043

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2022.

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If $A = \begin{bmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ then find the eigen values of A^{-1}.

2. Prove that $x^2 - y^2 + 4z^2 + 4xy + 2yz + 6xz$ is indefinite.

3. Evaluate : $\lim_{x \to 5} (2x^2 - 3x + 4)$.

4. Find the domain of the function $f(x) = \frac{2x^3 - 5}{x^2 + x - 6}$.

5. Prove $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$ if $f = x^3 + y^3 + z^3 + 3xyz$.

6. If $z = x^2 + y^2$, and $x = t^2$, $y = 2at$, find $\frac{dz}{dt}$.

7. Evaluate : $\int_0^{\pi/2} \sin^6 x \, dx$.

8. Prove that $\int_0^a f(x) dx = \int_0^a f(a-x) dx$.

9. Evaluate : $\int_1^2\int_1^3 xy^2\, dxdy$.

10. Evaluate : $\int_0^1\int_0^2\int_0^3 xyz\, dxdy\, dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find A^4 if $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form through an orthogonal reduction. (16)

12. (a) (i) For what values of a and b, is $f(x) = \begin{cases} -2, & x \le -1 \\ ax - b, & -1 < x < 1 \\ 3, & x \ge 1 \end{cases}$ continuous at every x? (8)

(ii) Find the differential coefficients of $\frac{(a-x)^2(b-x)^3}{(c-2x)^3}$. (8)

Or

(b) (i) Evaluate (1) $\frac{d}{dx}\left(3x^5 \log x\right)$ and (2) $\frac{d}{dx}\left(\frac{x^3}{3x-2}\right)$. (4+4)

(ii) Find the maximum and minimum values of $2x^3 - 3x^2 - 36x + 10$. (8)

13. (a) (i) If $x = u\cos v$ and $y = u \sin v$, prove that $\frac{\partial(x,y)}{\partial(u,v)} \times \frac{\partial(u,v)}{\partial(x,y)} = 1$. (8)

(ii) Obtain the Taylor's series expansion of $e^x \log(1+y)$ at the orign. (8)

Or

(b) (i) If $u = \log\left(\frac{x^5 + y^5}{x^3 + y^3}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2$. (8)

(ii) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. (8)

14. (a) (i) Evaluate $\int \frac{x + \sin x}{1 + \cos x} dx$. (8)

(ii) Use partial fraction technique, evaluate $\int \frac{3x+1}{(x-1)^2(x+3)} dx$. (8)

Or

(b) (i) Evaluate $\int_0^{\pi/4} \log(1 + \tan\theta) d\theta$. (8)

(ii) Find the mass M and the center of mass $\bar{x}$ of a rod lying on the x-axis over the interval $[1,2]$ whose density function is given by $\delta(x) = 2 + 3x^2$. (8)

15. (a) (i) Change the order of the integration $\int_0^\infty \int_x^\infty \frac{e^{-y}}{y} dydx$ and evaluate the same. (8)

(ii) Find the area of the region R enclosed by the parabola $y = x^2$ and the line $y = x + 2$. (8)

Or

(b) (i) Using polar coordinates, evaluate $\iint_R e^{x^2+y^2}\, dydx$, where R is the semicircular region bounded by the x-axis and the curve $y = \sqrt{1 - x^2}$. (8)

(ii) Calculate the volume of the solid bounded by the planes $x = 0$, $y = 0$, $z = 0$, and $x + y + z = 1$. (8)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 70132

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2022

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. The eigenvalues and the corresponding eigenvectors of a 2 × 2 matrix is given by $\lambda_1 = 8$; $x_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ and $\lambda_2 = 4$; $x_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$. Find the corresponding matrix.

2. Determine the nature, index and signature of the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_2x_3 + 6x_3x_1 + 2x_1x_2$.

3. For what values of the constant c is the function f continuous on $(-\infty, \infty)$?

$$f(x) = \begin{cases} cx^2 + 2x; & x < 2 \\ x^3 - cx; & x \geq 2 \end{cases}.$$

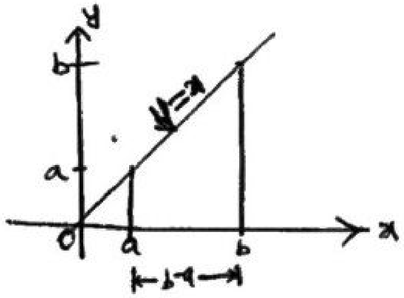
4. Find the slope of the circle $x^2 + y^2 = 25$ at $(3, -4)$.

5. Find $\frac{\partial^2 w}{\partial x \partial y}$, if $w = xy + \frac{e^y}{y^2 + 1}$.

6. Find $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x^2 + y^2$, $x = r - s$ and $y = r + s$.

7. Evaluate $\int \frac{\tan x}{\sec x + \tan x} dx$.

8. Find the area of the region shown in the diagram given below, bounded between $x = a$ and $x = b$.



9. Sketch the region of integration in $\int_0^1 \int_x^1 f(x, y)\,dy\,dx$.

10. Change the Cartesian integral $\int_0^6 \int_0^y x\,dx\,dy$ into an equivalent polar integral.

PART B — (5 × 16 = 80 marks)

11. (a) Obtain an orthogonal transformation which will transform the quadratic form $Q = 2x_2x_3 + 2x_3x_1 + 2x_1x_2$ to canonical form.

Or

(b) An elastic membrane in the x_1x_2-plane with boundary circle $x_1^2 + x_2^2 = 1$ is stretched so that a point $P = (x_1, x_2)$ goes over a point $Q = (y_1, y_2)$ given by $y_1 = 5x_1 + 3x_2$ and $y_2 = 3x_1 + 5x_2$. Find the principal directions that is, the directions of the position vector x of P for which the direction of the position vector y of Q is the same or exactly opposite. What shape does the boundary circle take under this deformation?

12. (a) (i) Find y'' if $x^4 + y^4 = 16$. (8)

(ii) Differentiate $y = (2x+1)^5 (x^3 - x + 1)^4$. (8)

Or

(b) Find the intervals on which $f(x) = -x^3 + 12x + 5;\ -3 \le x \le 3$ is increasing and decreasing. Where does the function assume extreme values? What are those values?

13. (a) Find the maximum and minimum values of the function $f(x,y)=3x+4y$ on the circle $x^2+y^2=1$.

Or

(b) Find the Taylor series expansion of the function $f(x,y)=\sin x \sin y$ near the origin.

14. (a) (i) Evaluate $\int_0^{\infty} e^{-ax} \sin bx dx$, for $a>0$. (8)

(ii) Integrate $\int_0^{\pi/2} \frac{\sin x \cos x}{\cos^2 x+3\cos x+2} dx$. (8)

Or

(b) (i) Evaluate $\int \frac{3x^4+3x^3-5x^2+x-1}{x^2+x-2} dx$. (8)

(ii) Integrate $\int x\sqrt{1+x-x^2}\, dx$. (8)

15. (a) (i) Change the order of integration in $\int_0^1 \int_{x^2}^{2-x} xy\, dy\, dx$ and hence evaluate. (8)

(ii) Find the area of the region inside the cardioid $r=a(1+\cos\theta)$ and outside the circle $r=a$. (8)

Or

(b) Find the volume of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=4$. (16)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 30234

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2023.

First Semester

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If two eigen values of the matrix $A=\begin{pmatrix}2 & 2 & 1\\1 & 3 & 1\\1 & 2 & 2\end{pmatrix}$ are equal to 1 each, find the eigen value of A^{-1}.

2. Write the uses of Cayley-Hamilton Theorem.

3. If $y=x\log\left(\frac{x-1}{x+1}\right)$, then find $\frac{dy}{dx}$.

4. Find the point of inflection of $f(x)=x^3-9x^2+7x-6$.

5. Write Euler's theorem on homogeneous functions.

6. If $x=r\cos\theta, y=r\sin\theta$, find $\frac{\partial(x, y)}{\partial(r,\theta)}$.

7. Evaluate $\int\theta\cos\theta\, d\theta$ using integration by parts.

8. Find the value of $\int_0^{\pi/2}\sin^6 x\, dx$.

9. Evaluate $\int_0^1\int_0^x dy\, dx$.

10. Transform the double integral $\int_0^2\int_y^2\frac{x\,dx\,dy}{x^2+y^2}$ into polar coordinates.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of $A=\begin{pmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{pmatrix}$. (8)

(ii) Verify Cayley-Hamilton theorem for the matrix $A=\begin{pmatrix}1 & 1 & 1\\ 1 & 2 & -3\\ 2 & -1 & 3\end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $2x_1x_2 - 2x_2x_3 + 2x_3x_1$ into the canonical form and hence find its nature. (16)

12. (a) (i) Find the values of a and b that make f continuous on $(-\infty, \infty)$ if

$$f(x)=\begin{cases} \dfrac{x^3-8}{x-2}, & if\ x<2 \\ ax^2-bx+3, & if\ 2\le x<3 \\ 2x-a+b, & if\ x\ge 3 \end{cases}$$ (8)

(ii) Find $\dfrac{dy}{dx}$ if $y=x^2e^{2x}\left(x^2+1\right)^4$. (4)

(iii) If $x^y=y^x$, Prove that $\dfrac{dy}{dx}=\dfrac{y(y-x\log y)}{x(x-y\log x)}$ using implicit differentiation. (4)

Or

(b) (i) Show that $\sin x(1+\cos x)$ is maximum when $x=\pi/3$. (6)

(ii) A window has the form of a rectangle surmounted by a semicircle. If the perimeter is 40 ft., find its dimensions so that greatest amount of light may be admitted. (10)

13. (a) (i) Given the transformations $u=e^x\cos y$ and $v=e^x\sin y$ and that f is a function of u and v and also of x and y, prove that $\dfrac{\partial^2 f}{\partial x^2}+\dfrac{\partial^2 f}{\partial y^2}=\left(u^2+v^2\right)\left(\dfrac{\partial^2 f}{\partial u^2}+\dfrac{\partial^2 f}{\partial v^2}\right)$. (8)

(ii) Expand $e^x\log(1+y)$ in powers of x and y up to terms of third degree. (8)

Or

(b) (i) Examine for extreme values of $f(x, y)x^4+y^4-2x^2+4xy-2y^2$. (8)

(ii) A rectangular box, open at the top, is to have a volume of 32 c.c. Find the dimensions of the box, that requires the least material for its construction. (8)

14. (a) (i) Evaluate $\int \frac{3x+1}{(x-1)^2(x+3)}dx$ by applying partial fraction on the integrand; (6)

(ii) Evaluate $\int_0^{\pi/2} \log \sin x \, dx$ and hence find the value of $\int_0^1 \frac{\sin^{-1} x}{x} dx$. (10)

Or

(b) (i) Evaluate $\int \frac{\sqrt{9-x^2}}{x^2} dx$ using trigonometric substitution. (6)

(ii) Determine whether the integral $\int_1^{\infty} \frac{1}{x} dx$ is convergent or divergent. (4)

(iii) Find the volume of the reel shaped solid formed by the revolution about the y-axis, of the part of the parabola $y^2 = 4ax$ cut off by its latusrectum. (6)

15. (a) (i) Find the area between the curves $y^2 = 4x$ and $x^2 = 4y$. (8)

(ii) Change the order of integration in $\int_0^{\infty}\int_0^{y} ye^{-y^2/x}\, dx\, dy$ and then evaluate it. (8)

Or

(b) (i) Find the volume of the sphere of radius 'a'. (8)

(ii) Find the moment of inertia of the area bounded by the curve $r^2 = a^2 \cos 2\theta$ about its axis. (8)

Reg. No. : E N G G T R E E . C O M

Question Paper Code : 21272

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2023.

First Semester

Civil Engineering

MA 3151 — MATRICES AND CALCULUS

(Common to : All Branches (Except Marine Engineering))

(Regulations 2021)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the eigenvalues of A^{-1} and A^2 if $A = \begin{pmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{pmatrix}$.

2. State Cayley-Hamilton theorem.

3. Sketch the graph of the function $f(x) = \begin{cases} x^2 & if\ -2 \le x \le 0 \\ 2-x & if\ 0 < x \le 2 \end{cases}$.

4. The equation of motion of a particle is given by $s = 2t^3 - 5t^2 + 3t + 4$ where s is measured in meters and t in seconds. Find the velocity and acceleration as functions of time.

5. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, find $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$.

6. Write any two properties of Jacobians.

7. Evaluate $\int_0^{\frac{\pi}{2}} \sin^9 x\, dx$.

8. Prove that the integral $\int_1^{\infty} \frac{1}{x} dx$ is divergent.

9. Evaluate $\int_1^2\int_1^3 xy^2\,dx\,dy$.

10. Find the area of a circle $x^2+y^2=a^2$ using polar coordinates in double integrals.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of $A=\begin{pmatrix}1 & 1 & 1\\ 0 & 2 & 1\\ -4 & 4 & 3\end{pmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find A^{-1} if $A=\begin{pmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $8x^2+7y^2+3z^2-12xy-8yz+4zx$ into the canonical form and hence find its rank, index, signature and nature. (16)

12. (a) (i) Let $f(x)=\begin{cases}\sqrt{-x} & if\ x<0\\ 3-x & if\ 0\le x\le 3\\ (x-3)^2 & if\ x>3\end{cases}$. Evaluate each of the following limits, if they exist.

(1) $\lim_{x\to 0^-} f(x)$

(2) $\lim_{x\to 0^+} f(x)$

(3) $\lim_{x\to 3^-} f(x)$

(4) $\lim_{x\to 3^+} f(x)$

(5) $\lim_{x\to 0} f(x)$

(6) $\lim_{x\to 3} f(x)$

Also, find where $f(x)$ is continuous. (8)

(ii) Find the n^{th} derivative of $f(x)=xe^x$. (4)

(iii) Differentiate $F(t)=\dfrac{t^2}{\sqrt{t^3+1}}$. (4)

Or

(b) (i) Use logarithmic differentiation to differentiate $y = \dfrac{x^{3/2}\sqrt{x^2+1}}{(3x+2)^5}$. (8)

(ii) Discuss the curve $f(x) = x^4 - 4x^3$ for points of inflection, and local maxima and minima. (8)

13. (a) (i) Given the transformations $u = e^x \cos y$ and $v = e^x \sin y$ and that f is a function of u and v and also of x and y, prove that

$$\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} = \left(u^2 + v^2\right)\left(\frac{\partial^2 f}{\partial u^2} + \frac{\partial^2 f}{\partial v^2}\right) \quad (8)$$

(ii) Expand $e^x \cos y$ in a series of powers of x and y as far as the terms of the third degree. (8)

Or

(b) (i) Examine for extreme values of $f(x, y) = x^3 + y^3 - 12x - 3y + 20$. (8)

(ii) A rectangular box, open at the top is constructed so as to have a volume of 108 cubic meters. Find the dimensions of the box that requires the least material for its construction. (8)

14. (a) (i) Find a reduction formula for $\int e^{ax} \sin^n x \, dx$. (8)

(ii) Integrate the following : $\int \dfrac{x^4 - 2x^2 + 4x + 1}{x^3 - x^2 - x + 1} dx$. (8)

Or

(b) (i) Evaluate $\int \sqrt{\dfrac{1-x}{1+x}} dx$. (8)

(ii) Find the centre of mass of a semicircular plate of radius r. (8)

15. (a) (i) Change the order of integration in $\int_0^4 \int_{x^2/4}^{2\sqrt{x}} xy \, dy \, dx$ and then evaluate it. (8)

(ii) Find the area enclosed by the curves $y = 2x - x^2$ and $x - y = 0$. (8)

Or

(b) (i) Find the volume of the tetrahedron bounded by the planes $x = 0$, $y = 0$, $z = 0$ and $\dfrac{x}{a} + \dfrac{y}{b} + \dfrac{z}{c} = 1$. (8)

(ii) Find the moment of inertia of a hollow sphere about a diameter, given that its internal and external radii are 4 meters and 5 meters respectively. (8)

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Reg. No. : | | | | | | | | | | | |

Question Paper Code : 51315

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2024.

First Semester

Civil Engineering

MA 3151 – MATRICES AND CALCULUS

(Common to : All Branches (Except B.E. Marine Engineering))

(Also Common to PTMA 3151-Matrices and calculus for B.E. (Part-Time) First Semester-All Branches-Regulations 2023)

(Regulations 2021)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If λ is an eigenvalue of a matrix A, then prove that λ^2 is an eigenvalue of A^2.

2. If $x = [-1, 0, 1]^T$ is the eigenvector of the matrix $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$, then find the corresponding eigen value.

3. Sketch the graph of the function $f(x) = 2.0 - 0.4x$ and find the domain of the function.

4. Differentiate $y = x\tan(\sqrt{x})$ with respect to x.

5. Verify Euler's theorem for the function $u = x^2 + y^2 + 2xy$.

6. If $u = x - y, v = y - z, w = z - x$, then find the Jacobian $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

7. What is wrong with the equation $\int_{-2}^{1}\left[\frac{1}{x^4}\right]dx = \int_{-2}^{1}\left[x^{-4}\right]dx = \left[\frac{x^{-3}}{-3}\right]_{-2}^{1} = -\frac{3}{8}$.

8. Evaluate $\int_{-1}^{1}\left[\frac{\tan x}{1 + x^2 + x^4}\right]dx$ by using the concept of odd and even functions.

9. Evaluate $\int_1^2 \int_0^{x^2} [x] dy\, dx$.

10. Write the integral equation for the regions $x \geq 0, y \geq 0, z \geq 0,\ x^2 + y^2 + z^2 \leq 1$ by triple integration.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the given matrix $A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & -1 \\ 0 & -1 & 3 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem, find the inverse of the given matrix $A = \begin{bmatrix} 1 & 0 & -1 \\ 2 & 1 & 1 \\ 1 & 0 & -2 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2 x_3 + 2x_3 x_1 - 2x_1 x_2$ to a canonical form by orthogonal reduction. (16)

12. (a) (i) Find the value of $\lim_{x \to 2} \left[\frac{x^2 - 2}{x^3 - 3x + 5} \right]^2$. (6)

(ii) Find the local maximum and minimum values of the function $f(x) = x + 2\sin x$ in the interval $0 \leq x \leq 2\pi$. (10)

Or

(b) (i) Find an equation of the tangent line to the curve $y = \frac{e^x}{(1 + x^2)}$ at the point $(1, e/2)$. (8)

(ii) Find the absolute maximum and absolute minimum values of the function $f(x) = \log\left[x^2 + x + 1\right]$ in the interval $[-1, 1]$. (8)

13. (a) (i) If $u = \log\left[x^2 + y^2 + z^2\right]$ then find the value of $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}$? (8)

(ii) The temperature at any point (x,y,z) in space is given by $T = 400xyz^2$. Find the maximum temperature on the surface of the unit sphere $x^2 + y^2 + z^2 = 1$. (8)

Or

(b) (i) Expand $f(x,y) = e^{x+y}$ about the point $(0,0)$ in powers of x and y upto third degree terms by using Taylor's series. (8)

(ii) Find the maxima and minima for the given function $f(x,y) = x^3y^2[1 - x - y]$. (8)

14. (a) (i) Evaluate $\int x^2 e^x \, dx$ by using integration by parts. (8)

(ii) Evaluate the integral $\int \sin^4 x \, dx$. (8)

Or

(b) (i) Evaluate $\int \sqrt{a^2 - x^2} \, dx$. (8)

(ii) Evaluate $\int \frac{1}{\left(x^2 - a^2\right)} dx$ by using partial fraction. (8)

15. (a) (i) Evaluate $\int_0^{\pi/2} \int_0^{\sin\theta} [r] d\theta \, dr$. (8)

(ii) Change the order of integration in

$\int_0^a \int_x^a \left[x^2 + y^2\right] dy \, dx$ and hence evaluate it. (8)

Or

(b) (i) Evaluate $\iint [xy] dx \, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. (8)

(ii) Find the volume of the sphere $x^2 + y^2 + z^2 = 3^2$ by using triple integration. (8)
