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Department of Electronics and Communication Engineering

EC3354 Signals and Systems

Unit 2 – ANALYSIS OF CONTINUOUS TIME SIGNALS

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Introduction to Signal Analysis

Signal analysis techniques are essential for transforming signals into frequency domain representations.

Three important techniques:

- **Fourier Series**: Used for periodic signals.
- **Fourier Transform**: Used for both periodic and non-periodic signals.
- **Laplace Transform**: Widely used for analyzing system behavior in the s-domain.

Fourier Series for Periodic Signals

- Fourier Series expresses a periodic signal as a sum of sinusoidal functions (sines and cosines).
- **Mathematical Expression:**

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

Where C_n are the Fourier coefficients and ω_0 is the fundamental angular frequency.

- Fourier series works for periodic signals with a period $T_0 = \frac{2\pi}{\omega_0}$.

Fourier Series Coefficients $C_n = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-jn\omega_0 t} dt$

- The Fourier coefficients C_n describe the amplitude and phase of each harmonic component of the signal.

Fourier Transform

- Fourier Transform is a generalization of the Fourier Series to non-periodic signals.
- Definition:

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft} dt$$

Where $X(f)$ is the frequency-domain representation of $x(t)$.

Example: Transform a simple signal (e.g., a Gaussian pulse) from the time domain to the frequency domain.

Properties of Fourier Transform

Linearity: The Fourier transform of a sum of signals is the sum of their Fourier transforms.

Time Shifting: Shifting a signal in time results in a phase shift in the frequency domain.

Frequency Shifting: Multiplying a signal by a complex exponential corresponds to a shift in the frequency domain.

Scaling: Compressing a signal in time causes expansion in frequency and vice versa.

Convolution: The Fourier transform of the convolution of two signals is the product of their Fourier transforms.

Laplace Transform

- Laplace Transform is a more general transform than the Fourier transform and is used to analyze systems in the complex s-domain.
- **Definition:**

$$X(s) = \int_0^{\infty} x(t)e^{-st}dt$$

Where $s = \sigma + j\omega$ is a complex variable.

Properties of Laplace Transform

- **Linearity**: The Laplace transform of a sum of functions is the sum of their Laplace transforms.
- **Time Shifting**: A shift in time corresponds to a multiplication by e^−as in the Laplace domain.
- **Frequency Shifting**: A frequency shift corresponds to e^{bs}.
- **Differentiation in Time Domain**: Differentiating a signal in time domain corresponds to multiplying its Laplace transform by s.
- **Initial and Final Value Theorems**: Provides the initial and final values of a signal based on its Laplace transform.

Fourier Transform vs Laplace Transform

- **Fourier Transform**: Primarily used for signals with known periodic or asymptotic behavior.
- Works with the $j\omega$ axis (frequency).
- **Laplace Transform**: Used for more general signals (including unstable or exponential signals).
- Works in the complex s-domain, enabling analysis of system stability and transient responses.

Applications in Signal Processing

- Fourier Series: Used in analyzing periodic signals, such as those in communication systems.
- Fourier Transform: Used in spectrum analysis, audio signal processing, and filter design.
- Laplace Transform: Used in control systems, stability analysis, and circuit analysis.

Example - Fourier Transform of a Signal

- Example: Compute the Fourier transform of a simple exponential decay signal.
- **Mathematical Expression:**

$$x(t) = e^{-\alpha t}u(t)$$

Where $u(t)$ is the unit step function.

- **Solution:**

$$X(s) = \frac{1}{s + \alpha}$$

Summary

- Fourier Series represents periodic signals as sums of harmonics.
- Fourier Transform generalizes this to both periodic and aperiodic signals.
- Laplace Transform provides a broader analysis for system behavior, particularly in the s-domain.

References

- Text books

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  - Proakis, J.G., "Digital Signal Processing", Pearson.
  - Haykin, S., Van Veen, B., "Signals and Systems", Wiley.