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Department of Electronics and Communication Engineering

EC3492 Digital Signal Processing

Unit I - Discrete Fourier Transform (DFT)

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Introduction to Discrete Fourier Transform (DFT)

DFT Overview:

The Discrete Fourier Transform (DFT) is a mathematical technique used to analyze the frequency content of discrete-time signals.

DFT converts a sequence of time-domain samples into a sequence of frequency-domain components.

Importance:

Used in applications such as signal processing, image processing, and data compression.

Sampling Theorem and Frequency in Discrete-Time Signals

Sampling Theorem:

A continuous-time signal can be completely represented by its samples if the sampling rate is at least twice the highest frequency present in the signal (Nyquist rate).

Discrete-Time Signal Frequency:

In discrete-time signals, frequency is expressed in terms of the sampling rate and the discrete frequency index.

The frequency spectrum of a discrete-time signal is periodic, with a period equal to the sampling rate.

Fourier Transform and DTFT

- **Fourier Transform (FT):**
  - Continuous transformation that maps time-domain signals to the frequency domain.
- **Discrete-Time Fourier Transform (DTFT):**
  - The DTFT is used to represent a discrete-time signal in the frequency domain.
  - The DTFT is continuous, while the DFT is discrete.

DTFT Equation:

$$X(f) = \sum_{n=-\infty}^{\infty} x(n) e^{-j2\pi f n}$$

- **DTFT to DFT Derivation:**
  - DFT is derived from DTFT by sampling the DTFT at discrete frequency points.

Discrete Fourier Transform (DFT)

- **DFT Definition:**
  - The DFT converts a finite sequence of N time-domain samples into an N-point sequence of frequency-domain components.
  - **DFT Equation:**

$$X(k) = \sum_{n=0}^{N-1} x(n) e^{-j\frac{2\pi kn}{N}}$$

where $k = 0, 1, 2, \ldots, N-1$ represents the frequency bins.

- **Frequency Domain Sampling:**
  - The DFT samples the frequency spectrum of the discrete signal at equally spaced intervals.

Properties of DFT

- **Periodicity:**
  - The DFT of a sequence is periodic with a period equal to N, the length of the sequence:

$$X(k) = X(k+N)$$

- **Symmetry:**
  - If the input signal is real, the DFT will exhibit conjugate symmetry:

$$X(N-k) = X^*(k)$$

- **Circular Convolution:**
  - DFT operates under the assumption of periodicity, leading to circular convolution when performing linear convolution.

Linear Filtering Using DFT

- Linear Filtering:
  - The DFT allows us to perform linear filtering by multiplying the frequency-domain representation of the signal with the frequency-domain representation of the filter.
  - DFT Filtering Equation:

$$Y(k) = X(k) \cdot H(k)$$

where Y(k) is the output, X(k) is the input, and H(k) is the frequency response of the filter.

Filtering Long Data Sequences

Challenges with Long Data:

The computational complexity of DFT increases for large data sequences, as it requires $O(N^2)$ operations.

Overlap-Add and Overlap-Save Methods:

These methods break long data sequences into smaller chunks to reduce computational load:

- **Overlap-Add**: Add overlapping segments of the signal.
- **Overlap-Save**: Save the last part of the segment to avoid boundary effects.

Both methods use the DFT for efficient filtering.

Fast Computation of DFT

- **Fast Fourier Transform (FFT):**
  - The FFT is an optimized algorithm to compute the DFT with a reduced computational complexity of O(N log N).
- **Radix-2 Decimation-in-Time (DIT) FFT:**
  - The Radix-2 FFT is an efficient algorithm for computing the DFT by recursively breaking the DFT into smaller DFTs.
- **Decimation-in-Frequency (DIF) FFT:**
  - An alternative to the DIT FFT, where the input sequence is recursively divided by frequency instead of time.

Linear Filtering Using FFT

FFT for Filtering:

By using FFT, we can efficiently perform filtering in the frequency domain for long data sequences.

FFT reduces the number of operations required for linear filtering, making it more efficient for real-time processing.

Summary of DFT

The DFT is a key tool for analyzing and processing discrete-time signals in the frequency domain.

Key concepts include:

DFT Equation and **Frequency Domain Sampling**

Properties: Periodicity, Symmetry, Circular Convolution

Applications: Linear filtering using DFT, FFT for fast computation, and methods for filtering long sequences.

References

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